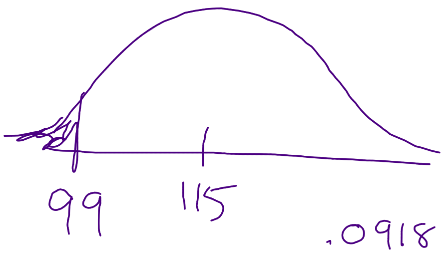
Review Unit IV

Name:____________________

1. Suppose the IQ scores of a sample of college students follow a normal distribution with a mean of 115 and a standard deviation of 12.
Show all work and include sketches.

a) What is the proportion of students that have an IQ score less than 99?



$z = \frac{99-115}{12} = -1.33$

$Pr(z < -1.33) = \boxed{.0918}$

.0918 students have an IQ score less than 99.

b) What is the proportion of students that have an IQ score between 105 and 120?

$z = \frac{105-115}{12} = -.83$ $Pr(z < -.83) = .2033$

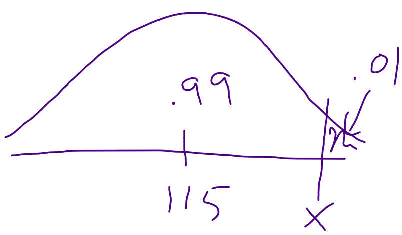
$z = \frac{120-115}{12} = .42$ $Pr(z < .42) = .6628$

.4595 of the students would . . .

$.6628 - .2033 = \boxed{.4595}$

$Pr(-.83 < z < .42)$

c) Determine how high one's IQ must be to be in the top 1% of all IQs at this college.



innorm(.99) = 2.33

$2.33 = \frac{x-115}{12}$

$x = 142.96$

A person would have to have a 143 IQ to be in the top 1%.

2. Complete the following chart with the appropriate symbols:

	Population Parameter	Sample Statistic
Proportion	p	$\hat{p}$
Mean	μ	$\bar{x}$
Standard Deviation	σ	s_x

3. **Consider the ESP testing where a subject is asked to guess which of the four cards is chosen. Probability of guessing correct is ¼. ($p = 0.25$) Each subject is presented with 40 cards. (n=40)**

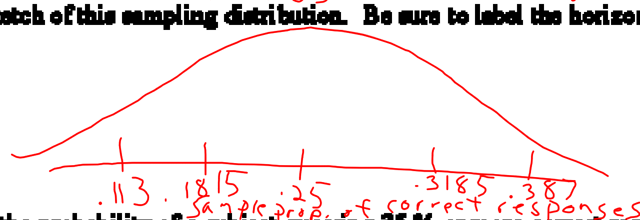
   a) **If a subject is just guessing what does the Central Limit Theorem say about the sampling distribution of the sample proportion of correct responses? Report the mean and standard deviation of this distribution and describe its shape as well. (Be sure to check the assumptions for CLT also.)**

The CLT says The samp. dist. of $\hat{p}$ of correct responses will be approx. normal because n is large: $40(.25) \geq 10$ $\quad 10 \geq 10$ $\quad 40(1-.25) \geq 10$ $\quad 30 \geq 10$

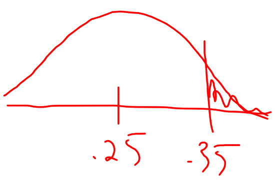
with a mean of $\hat{p}$ at .25 ($\mu_{\hat{p}} = .25$)

and a st. dev. of .0685 → $\sqrt{\frac{.25(1-.25)}{40}} = \sigma_{\hat{p}}$

   b) **Draw a sketch of this sampling distribution. Be sure to label the horizontal axis.**



   c) **Calculate the probability of a subject guessing 35 % or more correct responses.**



$$z = \frac{.35 - .25}{.0685} = 1.46 \qquad Pr(z > 1.46) = \boxed{.0721}$$

   d) **If the number of cards presented to each subject is increased to 160, what changes will occur in question c)? Explain, as well as find the new probability.**

$\sqrt{\frac{.25(1-.25)}{160}}$ ↓ $\sigma_{\hat{p}} = .0342$

the st. dev. of samp. dist. would decrease (cut in ½) which would decrease the prob. of guessing 35% or more correctly

$z = 2.92 \quad Pr(z > 2.92) = .0018$

   e) **Based on your answers to c) and d), in which case would it be more surprising if the subject got more than 35% correct? What conclusion could you make about this subject?**

n = 160 . . .

not guessing

4. Consider the population of American households that purchase Christmas presents, and consider the variable "amount expected to be spent on Christmas presents as reported in late November." Suppose that this population has mean $\mu = \$850$ and standard deviation $\sigma = \$250$.

a) If a random sample of 5 households is selected, is it valid to use the Central Limit Theorem to describe the sampling distribution of the sample mean? What if a sample of 500 households is selected? Explain.

$n=5$ no, CLT doesn't apply $\Rightarrow n < 30$ AND pop. probably isn't normal

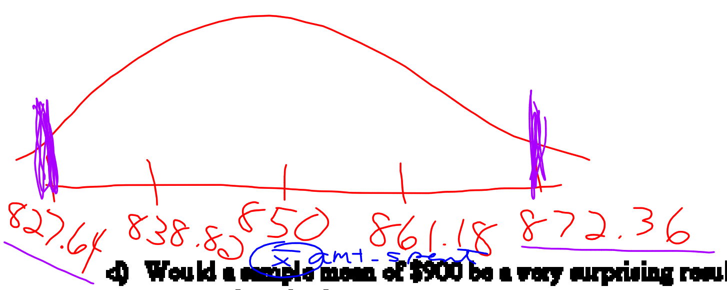
$n=500$ Yes, CLT applies: $n \geq 30$

b) What does the Central Limit Theorem say about how the sample mean would vary if samples of size 500 were taken over and over? Report the mean and standard deviation of this distribution and describe its shape as well.

The CLT says The samp. dist. of $\bar{x}$ would be:

- approx. normal $(500 \geq 30)$,
- with a mean of \$850 $(\mu_{\bar{x}} = 850)$
- and a st. dev. of $\frac{250}{\sqrt{500}} = 11.18 = \sigma_{\bar{x}}$

c) Draw a sketch of this sampling distribution. Be sure to label the horizontal axis.



d) Would a sample mean of \$900 be a very surprising result? Explain. (Include calculations to support your explanation.)

$z = \frac{900-850}{11.18} = 4.47$ $Pr(z > 4.47) \approx 0$

Yes $\Rightarrow$ very surprising

e) Continue to assume $\sigma = 250$, but return to not knowing the value of μ. Using a sample result of $\bar{x} = \$900$, for a sample size of 500, create an interval that contains 95% of the possible parameter values that could have generated this sample mean.

$\bar{x}$

$900 \pm 2(11.18)$ 2 st. dev.

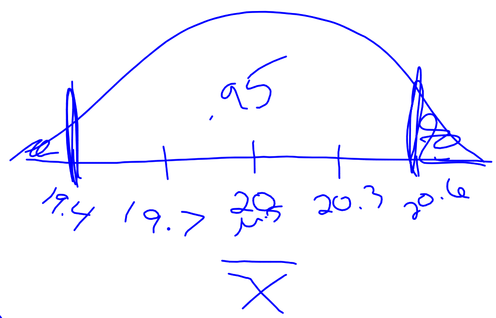
$(877.64, 922.36)$

$\mu = 20$

$\sigma = 3$

$n = 100$

$\sigma_{\bar{x}} = \frac{3}{\sqrt{100}} = .3$



What values of $\bar{x}$ would be surprising?

95% of $\bar{x}$ are w/in 2 st. dev.

(19.4, 20.6)

below

above

	Coeff	~~SE~~	~~t~~	~~p~~
Constant	−296.0707			
X(yrs)	111.8369			

~~SE~~ r-sq = .9095 ~~r-sq(adj)~~

$\hat{y} = a + bx$

variables: x, y

coeff: a, b

$\hat{y} = -296.0707 + 111.8369x$

a) corr. coeff = r

$r = \sqrt{.9095} = \boxed{.9537}$

strength
direction
linear assoc.
between x & y (in context)

There's a strong, pos, linear assoc. between price & life exp. of fridge.

b) slope → the predicted inc/dec in y for each additional inc. of x.

* The price is predicted to inc. $111.84 for each additional yr.

* y-int: predicted value of y when x = 0.
Predicted price is −$296.07 for a fridge w/ a 0 yr. life exp.

c) $\hat{y} = -296.0707 + 111.8369(10)$ (y = $750)

$\hat{y} = \$822.29$

$y - \hat{y}$

Residual = actual − pred.
= 750 − 822.29
= −72.29

d) infl. obs. → extreme x-value (35 yr.)
pull LSRL down
make slope less steep (decrease)

* outlier: ex (15, $250)